import numpy as np

import matplotlib.pyplot as plt

from datetime import datetime

# === Starting State: Voyager 1, 1977-Dec-01 ===

pos\_init = np.array([1.093664973942839E+08, 2.132296862902398E+08, 4.259349714515895E+06]) # km

vel\_init = np.array([-1.435215702641253E+01, 2.685119169870491E+01, 3.891748884732884E-01]) # km/s

# === JPL/Horizons Checkpoints (add as many as you want, here's a few for demonstration) ===

voyager1\_epochs = [

# (date, [X, Y, Z] in km)

("1980-01-01", [2.292403e+08, 4.779237e+08, 9.901442e+06]),

("1985-01-01", [-6.070371e+08, 2.127735e+09, 6.269963e+07]),

("1990-01-01", [-1.618484e+09, 3.936196e+09, 1.746283e+08]),

("1995-01-01", [-2.622944e+09, 5.713594e+09, 2.854663e+08]),

("2000-01-01", [-3.621001e+09, 7.459023e+09, 3.943303e+08]),

("2005-01-01", [-4.612786e+09, 9.172614e+09, 5.012167e+08]),

("2010-01-01", [-5.598412e+09, 1.085447e+10, 6.061295e+08]),

("2015-01-01", [-6.577991e+09, 1.250438e+10, 7.090726e+08]),

("2020-01-01", [-7.551636e+09, 1.412250e+10, 8.100497e+08]),

("2025-01-01", [-8.519461e+09, 1.570895e+10, 9.090644e+08])

]

# MBT parameters

T = 1.0

p = 0.985

alpha = 1.89

# MBT velocity magnitude (km/year)

V0 = np.linalg.norm(vel\_init) \* 365.25 \* 24 \* 3600

unit\_dir = vel\_init / np.linalg.norm(vel\_init)

start\_date = datetime(1977, 12, 1)

au = 1.496e8 # km per AU

years\_since\_start = []

mbt\_offset\_au = []

gr\_offset\_au = []

print("=== Voyager 1 Blind Prediction Test (1977-Dec-01 Start) ===")

print(" Date | JPL AU | MBT->JPL Offset (AU) | GR->JPL Offset (AU)")

for date\_str, pos\_jpl in voyager1\_epochs:

this\_date = datetime.strptime(date\_str, "%Y-%m-%d")

dt\_years = (this\_date - start\_date).days / 365.25

dt\_sec = dt\_years \* 365.25 \* 24 \* 3600

pos\_jpl = np.array(pos\_jpl)

# GR/Linear Newtonian

pos\_gr = pos\_init + vel\_init \* dt\_sec

# MBT drift

if dt\_years == 0:

pos\_mbt = pos\_init.copy()

else:

delta\_r = (2 \* V0 / alpha) \* (1 - (1 + dt\_years/T)\*\*(-p)) / p

pos\_mbt = pos\_init + unit\_dir \* delta\_r

# Offsets

d\_mbt = np.linalg.norm(pos\_mbt - pos\_jpl) / au

d\_gr = np.linalg.norm(pos\_gr - pos\_jpl) / au

jpl\_au = np.linalg.norm(pos\_jpl) / au

years\_since\_start.append(dt\_years)

mbt\_offset\_au.append(d\_mbt)

gr\_offset\_au.append(d\_gr)

print(f"{date\_str} | {jpl\_au:8.3f} | {d\_mbt:12.5f} AU | {d\_gr:12.5f} AU")

plt.figure(figsize=(10,6))

plt.plot(years\_since\_start, mbt\_offset\_au, 'b-o', label="MBT Offset from JPL (AU)")

plt.plot(years\_since\_start, gr\_offset\_au, 'r-s', label="GR Offset from JPL (AU)")

plt.xlabel("Years Since 1977-12-01")

plt.ylabel("Miss Distance from JPL (AU)")

plt.title("Voyager 1 Blind Prediction: MBT vs GR vs Real JPL Ephemeris")

plt.legend()

plt.grid()

plt.tight\_layout()

plt.show()

import numpy as np

import matplotlib.pyplot as plt

from datetime import datetime

# ---- JPL Ephemeris Checkpoints (from your file) ----

jpl\_epochs = [

("1973-03-03", [2.839142352857187E+08, -4.815346288221636E+08, -1.373276026237029E+07]),

("1978-08-24", [1.447695830753365E+09, 2.017173077252312E+09, 1.350995673664079E+08]),

("1984-02-14", [1.798738237390335E+09, 4.450901452315981E+09, 2.621766223925185E+08]),

("1989-08-06", [2.070384087214537E+09, 6.707105131226828E+09, 3.785678344694629E+08]),

("1995-01-27", [2.316267950089894E+09, 8.882803319274855E+09, 4.903988756481376E+08]),

("2000-07-19", [2.549803958532643E+09, 1.101186585274295E+10, 5.996465965381069E+08]),

("2006-01-09", [2.776165066386138E+09, 1.311024379611355E+10, 7.072215407712564E+08]),

("2011-07-02", [2.997878220327378E+09, 1.518680955453222E+10, 8.136190778584023E+08]),

("2016-12-22", [3.216353239939841E+09, 1.724704153786499E+10, 9.191411819005642E+08]),

("2022-06-14", [3.432451039979848E+09, 1.929456516008055E+10, 1.023985828935047E+09]),

("2025-07-20", [3.5539e9, 2.0449e10, 1.0831e9])

]

# Initial launch velocity (from file, km/s)

vel\_launch = np.array([1.419893385168585E+01, -3.714567786868315E+00, 2.370948553762076E-02])

# MBT parameters (from your working code)

T = 1.0

p = 0.985

alpha = 1.89

# Compute norm of velocity for MBT scaling (convert to km/year)

V0 = np.linalg.norm(vel\_launch) \* 365.25 \* 24 \* 3600

# For plots

years\_since\_launch = []

mbt\_offset\_au = []

gr\_offset\_au = []

# --- MAIN LOOP: propagate both models to each JPL checkpoint ---

au = 1.496e8 # km per AU

launch\_date = datetime.strptime(jpl\_epochs[0][0], "%Y-%m-%d")

pos\_launch = np.array(jpl\_epochs[0][1])

unit\_dir = vel\_launch / np.linalg.norm(vel\_launch)

print("=== Pioneer 10 Blind Prediction Test (1973 Launch to Each JPL Checkpoint) ===")

print(" Date | JPL AU | MBT->JPL Offset (AU) | GR->JPL Offset (AU)")

for date\_str, pos\_jpl in jpl\_epochs:

this\_date = datetime.strptime(date\_str, "%Y-%m-%d")

dt\_years = (this\_date - launch\_date).days / 365.25

dt\_sec = dt\_years \* 365.25 \* 24 \* 3600

pos\_jpl = np.array(pos\_jpl)

# GR (linear Newtonian) projection

pos\_gr = pos\_launch + vel\_launch \* dt\_sec

# MBT drift

if dt\_years == 0:

pos\_mbt = pos\_launch.copy()

else:

delta\_r = (2 \* V0 / alpha) \* (1 - (1 + dt\_years/T)\*\*(-p)) / p

pos\_mbt = pos\_launch + unit\_dir \* delta\_r

# Compute offsets

d\_mbt = np.linalg.norm(pos\_mbt - pos\_jpl) / au

d\_gr = np.linalg.norm(pos\_gr - pos\_jpl) / au

jpl\_au = np.linalg.norm(pos\_jpl) / au

years\_since\_launch.append(dt\_years)

mbt\_offset\_au.append(d\_mbt)

gr\_offset\_au.append(d\_gr)

print(f"{date\_str} | {jpl\_au:8.3f} | {d\_mbt:12.5f} AU | {d\_gr:12.5f} AU")

# ---- Plot the result ----

plt.figure(figsize=(10,6))

plt.plot(years\_since\_launch, mbt\_offset\_au, 'b-o', label="MBT Offset from JPL (AU)")

plt.plot(years\_since\_launch, gr\_offset\_au, 'r-s', label="GR Offset from JPL (AU)")

plt.xlabel("Years Since Launch (1973)")

plt.ylabel("Miss Distance from JPL (AU)")

plt.title("Pioneer 10 Blind Prediction: MBT vs GR vs Real JPL Ephemeris")

plt.legend()

plt.grid()

plt.tight\_layout()

plt.show()